Control of a high Precision Double Drive

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The speed control system for a concept for cost effective drives with high precision is presented. The drive concept consists of two parallel working drives. The concept is an alternative to direct drives. One big advantage is the use of standard gear boxes with economical components. This paper deals with the control of the drive system consisting of two parts: one drive produces the power for the machine, another drive makes the motion precice and dynamic. Both drives are combined to one double drive by a control system. The drive system is usefull for printing machines and other machines with high power consumption at a nearly constant speed and high accuracy requirements.

The calculation for a drive system with 37 kW shows, that the control drive has to supply only about 20 % of the total torque and power needed to compensate the errors of the power drive. The stability of the system is shown by a simulation of the double drive.

1 Introduction

Modern production machines often need high precision rotations at high speed and power above 20 kW. The actual energy solving strategies need drives with overall good efficiency combined with high accuracy. E.g. printing machines have many printing cylinders with high requirements of accuracy and efficiency. The different cylinders have to rotate with minmal angle error in relation to a set angle. Other examples are punching machines. There drums have to work with precision and accuracy.

Nowadays such requirements need powerfull drives with both, high precision and high efficiency. This leads to very expensive drives.

In this paper a concept with a double drive and its control for high precision drives for production machines is shown. This concept is a cost effective alternative to used solutions in the power range above 20 kW. The basic idea is as follows: one drive with high power supports the machine with the main part of the power. A second drive with low power is responsible for the precision of the whole drive system. Figure 1 shows this Idea for a combination of a geared motor for the power drive PD and a direct coupled servo drive for the precision. The idea of a double drive is presented in [3, 4]. On the following pages the control and stabilization of the drive is examined.

Due to this idea the power drive can be optimized for high efficiency with acceptable costs. The control drive can be optimized for high precision. The efficiency and cost can be less good, because the small power of the control drive leads to acceptable losses and costs.

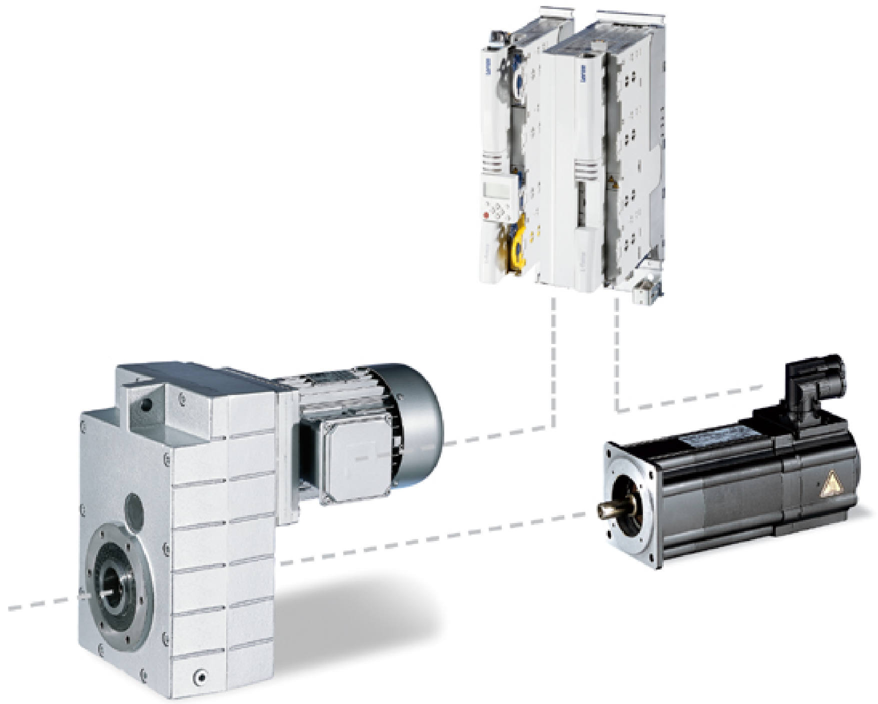

Figure 1: Double drive consisting of a geared motor with inverter and a servo motor with servo inverter (source: Lenze SE)

Precision in this context means

- High stiffness of the drive system against changes of the load and against disturbance
- fast control with high bandwidth
- small angle errors or distance errors e.g. infected by a backlash of a gear box or coupling

Due to the division of the tasks for power and precision to two different drives the power drive PD can be constructed cost optimized for the needed power, torque and speed, whereas the precice control drive CD only needs a small power to compensate the errors of the power drive.

The control for both drives makes the behavior of the complete double drive precise and linear. The double ndrive is integrated into an overlaying control system e.g. angle control for synchronizing printing marks.

The double-drive ist illustrated at a drum drive with gearbox and backlash (figure 2). The power drive PD consists of a standard asynchronous motor with a gear box and a frequency inverter. The gear box is connected to the load by a shaft with the stiffness C_{PD} . An angle sensor is mounted to the shaft of the gear box. The control drive CD is connected to the load by a shaft too. This shaft has the stiffness C_{CD} .

The set value for the control drive CD is the difference of set speed $\omega_{set} = n_{set}$ for the complete drive and the actual speed of the power Drive $\omega_{CD} = n_{CD}$ from the angle and speed sensor at the gear box multiplied with the gain V .

The values of the drives are shown in table 1.

Table 1: Data of the drives including control values

part of the drive	symbol	value
rated values of the power drive PD/ the complete drive		
speed	n_{PD}	180 ^{1}/min = 3 ^{1}/s
power	P_{PD}	37 kW
torque	M_{PD}	1963 Nm
errors of the power drive PD		
time constant of the power drive	T_{PD}	0.03183 s
gain of the power drive	V_{PD}	3000 $\frac{\text{Nm s}}{\text{rad}}$
torque deas zone of the power drive	M_{dz}	±100 Nm
sinusoidal torque distortion of the power drive	$\hat{M}_{sin}$	150 Nm
	f_{sin}	6 Hz
angle error due to eccentricity	e	50 µm
	d	120 mm
	α	30°
	$\Delta\varphi_{eccentricity}$	1.5 arcmin
backlash of gear box	$\Delta\varphi_{backlash}$	15 arcmin
inertias, stiffnesses		
inertia of the power drive PD	$J_{PD} = i^2 \cdot J^*_{PD}$	27.5 kg m^2
stiffness between load and power drive PD	C_{PD}	200 000 $\frac{\text{Nm}}{\text{rad}}$
inertia of the control drive CD	J_{CD}	0.032 kg m^2
stiffness between load and control drive CD	C_{CD}	50 000 $\frac{\text{Nm}}{\text{rad}}$
inertia of the load, neglected in the dynamic analysis	J	4 kg m^2
control drive CD		
gain of the control drive	C_{PD}	200 000 $\frac{\text{Nm}}{\text{rad}}$
time constant of the control drive	T_A	0.001 s
proportional part of the control drive	a_p	0.005 s
differential part of the control drive	a_d	−0.00005 s^2
minimum value of a_p according equations (77) and (78)		0.0015788 s
maximum value of a_p according equations (77) and (78)		0.0402667 s
maximum value of a_p according equation (74)		0.173275 s
minimum value of a_d according equation (64)		−0.00013766 s^2
maximum value of a_d according equations (71) and (72)		$-3.6441 \cdot 10^{-6}$ s^2
cutoff frequency according equation (52)	f_{maxCD}	15.91 Hz
cutoff frequency according equation (48)	f_{max}	833.6 Hz
torque and power for error correction		
constant torque error due to torque dead zone	$M_{CDconst}$	100 Nm
(equations (3) and (4)	$P_{CDconst}$	1885 W
sinusoidal torque error of power drive	$\hat{M}_{CDtorque}$	151.3 Nm
(equations (8) and (9)	$P_{CDtorque}$	2856 W
bandwidth of power drive	$\hat{M}_{CDbandwidth}$	140.5 Nm
(equations (17) and (18)	$P_{CDbandwidth}$	2890 W
angle error due to eccentricity of gear (equation (23))	$\hat{M}_{CDangle}$	2.1 Nm
backlash of gear (equation (26)	$\hat{M}_{CDbacklash}$	17.5 Nm
sum of torque (equation (1))	$\hat{M}_{CDcorrection}$	411.4 Nm

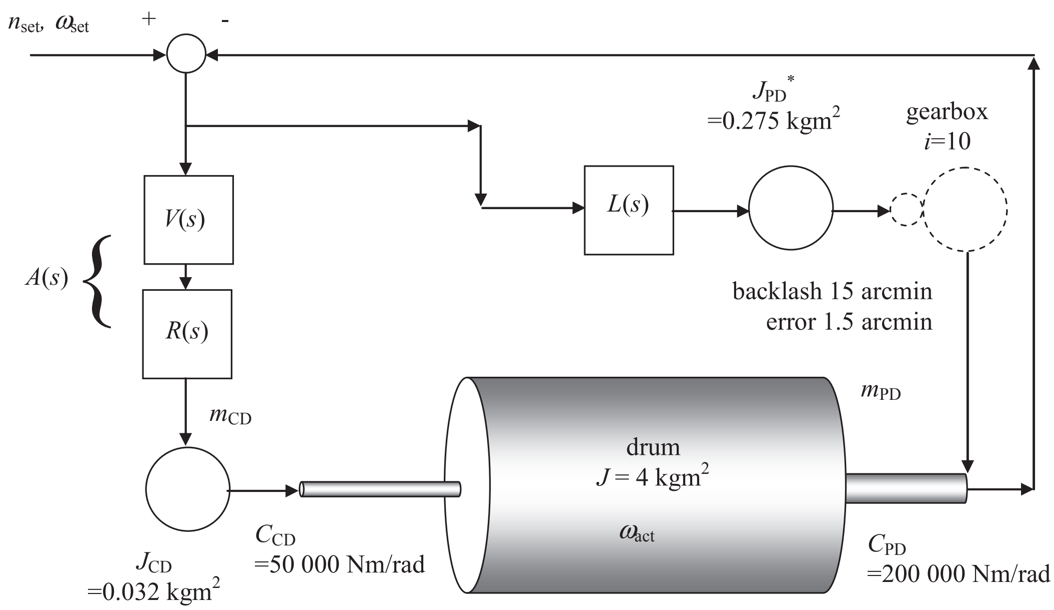

Figure 2: Double drive with drum and geared motor

2 Actual Solutions

Today in many applications electromechanical drives are used for high precision rotating applications. To supply the working machine with the requested low speed rotation often motor-gear-box combinations with planetary gear boxes with low backlash are used. The gear boxes are created for the high precision movement.

If an error caused by the mechanical construction due to a backlash of gears, elasticity of belts or chains is too big for the application direct drives are used for high precision movement.

The actual solutions have one common problem: the complete drive has to be built for the complete power and at the same time for the required accuracy. Table 2 shows measures to increase the accuracy in actual drives. All measures result in expensive drives.

The described measures are very expensive. Especially while operating with high power the components with standard accuracy are much cheaper than the high accuracy ones. Another problem is to produce the high precision components in a stable process. Often the high accuracy can only be manufactured by selecting suitable parts (e.g. gears) and combining them to a precice component (e.g. gear box). This is not usefull in practice for gear boxes with high torque and power.

In the following section a cost effective solution is described to get a high precision movement by combining two drives to a drive system: one drive for the power and one for the precision.

Table 2: Measures to increase the accuracy

Measure	Effect
low backlash gear box due to small tolerances in the gears	small angle error
coupling without backlash, e.g. (multiplate clutch, metal bellows)	small angle errors
couplings and gears with high stiffness, e.g. planetary gear, thick housings	small angle errors, higher gains
direct drives e.g. low speed synchronous machines	elimination of errors due to the mechanics

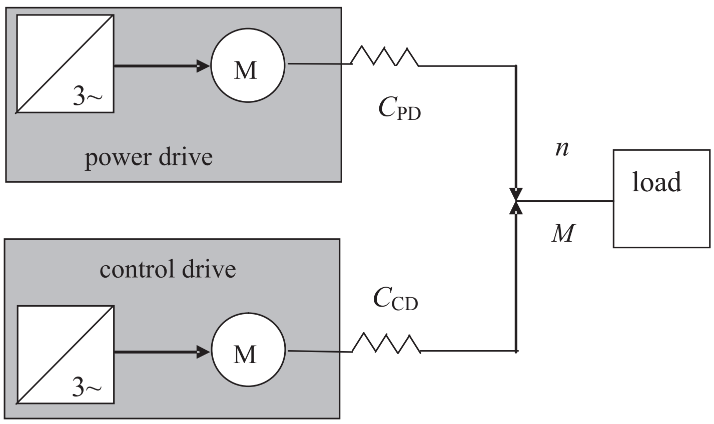

Figure 3: Scheme of double drive consisting of power drive PD and control drive CD

3 Double Drive with a Power Drive and a Control Drive

The complete drive is divided into two parts (figure 3). One drive (the power drive PD) supplies the machine with the required power. The second drive (the control drive CD) compensates the errors in movement produced by the power drive.

The power drive PD is optimized for the power production: The cost is low and the accuracy is less than the required accuracy. The disadvantages in accuracy are excepted to have advantages regarding in cost, weight and volume.

The control drive CD is made for high precision. E.g. the measures mentioned above offer a precise movement. The costs for these measures are acceptable for the control drive CD because the power supplied by the control drive is much lower than the complete power required.

Therefore the components for the control drive can be manufactured with low costs while achieving the required accuracy.

Later on an example for a double drive is presented. In this example with 37 kW overall power the power drive has to provide 37 kW and the control drive 7.6 kW. This means the control drive needs only

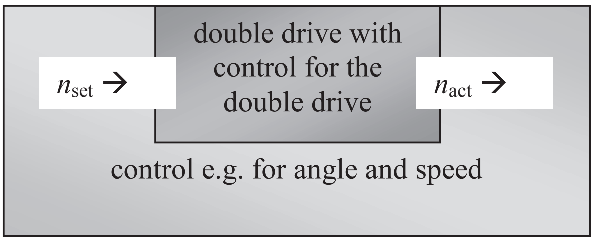

Figure 4: Integration of the double drive into an overlaying control system

about 20% of the total power. The biggest part of the power ist produced by the power drive PD.

Both drives are connected to the load by a common shaft and both together they produce the power that the load needs for a proper movement.

One control unit controls both drives (power drive PD and control drive CD) so that the double drive system works as one complete drive. The double drive is connected to an overlaying control system (figure 4).

4 Torque and Power needed to compensate Errors of the Power Drive

The control drive CD is supposed to compensate the errors of the power drive PD. Therefore the required power of the control drive depends on the errors of the power drive PD. The kind of error and its value guide to the power to compensate the error. On the following pages several kinds of errors and backlash of a gear are examined.

The total torque the control drive has to provide is

$$\boxed{\hat{M}_{\text{CDcorrection}} = M_{\text{CDconst}} + \hat{M}_{\text{CDtorque}} + \hat{M}_{\text{CDbandwidth}} + \hat{M}_{\text{CDangle}} + \hat{M}_{\text{CDbacklash}}} \tag{1}$$

For the example with $P_{\text{PD}} = 37\,\text{kW}$ at $n_{\text{PD}} = 180\,{}^{1}/\text{min}$ and $M_{\text{PD}} = 1963\,\text{Nm}$ the total torque of the control drive is (calculation see below):

$$\hat{M}_{\text{CDcorrection}} = (100 + 151.3 + 140.5 + 2.1 + 17.5)\ \text{Nm} = 411.4\,\text{Nm} \tag{2}$$

This is only about 20% of the torque M_{PD} of the power drive. Therefore the control drive can be made with low cost.

4.1 Torque and Power needed to compensate a Torque Error of the Power Drive

In the first step torque errors of the power drive PD are examined. These errors can occure in inverter drives due to electromagnetic differences in the power drive or to differences between the set value and actual value of the drive. Another cause for torque deviations is a too small bandwith of the power drive in relation to the requirements of the machine.

Reasons for torque deviations:

- temperature coefficient of the current torque behavior
- differences between the real motor and the motor model in the motor control
- direct current in the three phase output of the frequency inverter
- unbalanced current in the three phase output of the frequency inverter
- to small bandwith of torque control or speed control

Both drives (figure 5) act with elastical shafts or couplings C_{PD} and C_{CD} on the load. Together they produce the torque M. The power drive PD produces the torque M_{PD} with the error ΔM_{PD}.

Three kinds of torque errors are treated in the following sections:

- constant torque error
- periodically torque error
- torque error due to small bandwith

4.1.1 Torque and Power needed to compensate a constant Torque Error of the Power Drive

If the torque error ΔM_{PD} is independend of the time the control drive CD has to compensate this error with the torque

$$\boxed{M_{CDconst} = \Delta M_{PD}} \tag{3}$$

and the power

$$\boxed{P_{CDconst} = 2\pi n_{PD} \Delta M_{PD}} \,. \tag{4}$$

For the example this is the dead zone ±100 Nm of the drive:

$$M_{CDconst} = 100\,\text{Nm} \tag{5}$$

$$P_{CDconst} = 1885\,\text{W} \tag{6}$$

4.1.2 Torque and Power needed to compensate a periodically Torque Error of the Power Drive

In many cases the torque error varies periodicaly. In this case the error approximately equals the following function:

$$\Delta M_{PD}(t) = \Delta \hat{M}_{PD} \cdot \sin(2\pi \cdot f_{\sin} \cdot t) \tag{7}$$

To compensate this periodically varying torque the control drive CD not only has to produce the torque $\Delta M_{PD}(t)$, but also the control drive CD has to turn the elasticity by an angle to transfer the torque to the load. This means that the control drive periodically has to accelerate and slow down. This requires an additional torque for the control drive. The torque for torque correction and acceleration and deceleration is as follows.

$$\boxed{\hat{M}_{\text{CDtorque}} = \Delta\hat{M}_{\text{PD}} + J_{\text{CD}}\,(2\pi f_{\sin})^2\,\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}} \tag{8}$$

The corresponding power of the control drive CD is

$$\boxed{\begin{aligned} P_{\text{CDtorque}} &= 2\pi\cdot\left(n_{\text{PD}} + f_{\sin}\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\cdot\left(\Delta\hat{M}_{\text{PD}} + J_{\text{CD}}\,(2\pi f_{\sin})^2\,\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\\ &= 2\pi\cdot\left(n_{\text{PD}} + f_{\text{set}}\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\cdot\hat{M}_{\text{CDtorque}}\end{aligned}} \tag{9}$$

For the examble described in table 1 the equations give following values for torque and power:

$$\hat{M}_{\text{CDtorque}} = \left(150 + 0.032\cdot(2\pi 6)^2\,\frac{150}{50000}\right)\text{Nm} = 151.3\,\text{Nm} \tag{10}$$

$$P_{\text{CDtorque}} = 2\pi\left(3 + 6\cdot\frac{150}{50000}\right)\cdot\left(150 + 0.032\,(2\pi 6)^2\,\frac{150}{50000}\right)\text{W} = 2856\,\text{W} \tag{11}$$

4.1.3 Torque and Power needed to compensate a Torque Error caused of to small Bandwith of the Power Drive

The power drive acts with a limited bandwidth. This leads to a deviation between set torque M_{set} and actual torque M_{act}. For a required cutoff frequency of $\omega_{\text{s}} = 2\pi f_{\text{s}}$ the set torque follows for a torque jump $\hat{M}_{\text{set}}$ the following function:

$$M_{\text{set}} = \hat{M}_{\text{set}}\cdot\left(1 - \text{e}^{-\omega_{\text{s}}\cdot t}\right) \tag{12}$$

The power drive with the cutoff frequency $\omega_{\text{PD}} = 2\pi f_{\text{PD}}$ serves the torque

$$M_{\text{PD}} = \hat{M}_{\text{set}}\cdot\left(1 - \text{e}^{-\omega_{\text{PD}}\cdot t}\right). \tag{13}$$

The difference between required torque and actual torque is

$$\Delta M = M_{\text{set}} - M_{\text{PD}} = \hat{M}_{\text{set}}\cdot\left(1 - \text{e}^{-\omega_{\text{s}}\cdot t}\right) - \hat{M}_{\text{set}}\cdot\left(1 - \text{e}^{-\omega_{\text{PD}}\cdot t}\right) = \hat{M}_{\text{set}}\cdot\left(\text{e}^{-\omega_{\text{PD}}\cdot t} - \text{e}^{-\omega_{\text{s}}\cdot t}\right). \tag{14}$$

The maximum difference $\Delta\hat{M}_{\text{PD}}$ is reached at the time t_{x}:

$$t_{\text{x}} = \frac{\ln\frac{\omega_{\text{s}}}{\omega_{\text{PD}}}}{\omega_{\text{s}} - \omega_{\text{PD}}} \tag{15}$$

$$\Delta\hat{M}_{\text{PD}} = \hat{M}_{\text{set}}\cdot\left(\text{e}^{-\omega_{\text{PD}}\cdot t_{\text{x}}} - \text{e}^{-\omega_{\text{s}}\cdot t_{\text{x}}}\right) \tag{16}$$

The control drive not only has to provide the torque $\Delta\hat{M}_{\text{PD}}$ to compensate this torque error. Additionally the control drive CD has to turn the elasticity by an angle to transfer the torque to the load. This is the same context as in section 4.1.2. Therefore the control drive has to produce the torque

$$\boxed{\hat{M}_{\text{CDbandwidth}} = \Delta\hat{M}_{\text{PD}} + J_{\text{CD}}\,(2\pi f_{\text{s}})^2\,\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}} \tag{17}$$

and the corresponding power

$$\boxed{\begin{aligned} P_{\text{CDbandwidth}} &= 2\pi\cdot\left(n_{\text{PD}}+f_{\text{s}}\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\cdot\left(\Delta\hat{M}_{\text{PD}}+J_{\text{CD}}\left(2\pi f_{\text{s}}\right)^2\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\\ &= 2\pi\cdot\left(n_{\text{PD}}+f_{\text{s}}\frac{\Delta\hat{M}_{\text{PD}}}{C_{\text{CD}}}\right)\cdot\hat{M}_{\text{CDbandwidth}}\end{aligned}} \tag{18}$$

For the example the maximum set torque is calculated from the acceleration to correct an angle:

$$\hat{M}_{\text{set}} = 2\pi\cdot\left(J_{\text{PD}}+J\right)\frac{\Delta n}{\Delta t} = 2\pi\cdot(27.5+4)\frac{\frac{3}{60}}{0.1}\,\text{Nm} = 99\,\text{Nm} \tag{19}$$

The time t_{x} is calculated from the cutoff frequencies $f_{\text{s}} = 159$ Hz and $f_{\text{PD}} = 5$ Hz to $t_{\text{x}} = 3.573$ ms. This leads to a torque error of

$$\Delta\hat{M}_{\text{PD}} = 85.7\,\text{Nm}. \tag{20}$$

The control drive has to provide the torque

$$\hat{M}_{\text{CDbandwidth}} = \left(85.7+0.0032\cdot(2\pi 159)^2\cdot\frac{85.7}{50000}\right)\text{Nm} = 140.5\,\text{Nm} \tag{21}$$

and the power

$$P_{\text{CDbandwidth}} = 2\pi\left(3+159\frac{85.7}{50000}\right)\cdot 140.5\,\text{W} = 2890\,\text{W} \tag{22}$$

4.2 Torque needed to compensate an Angle Error

Drives with gears have deviations from the ideal motion caused by deviations in the teeth and eccentricities in the gears and shafts of the transmission. This can be detected as an angular error between PD and load. As an example, consider a pinion with a pitch diameter of 120 mm and a pressure angle of 30°. An eccentricity of the pinion of 50µm results in a tangential error of $50\mu\text{m}\cdot\sin 30° = 25\mu\text{m}$. This results in an angular error of $\frac{\Delta\varphi}{2} = \frac{25\mu\text{m}}{120\text{mm}} = 1.5\,\text{arcmin}$. The error of $\Delta\varphi = 1.5\,\text{arcmin}$ must be compensated by the CD. This requires that the control drive CD speeds up the entire drive in order to eliminate the angle error. To simplify this problem the elasticity between the drive and load is not considered. The CD has to produce the torque

$$\boxed{\hat{M}_{\text{CDangle}} = J_{\text{PD}}\frac{\Delta\varphi}{2}(2\pi n)^2} \tag{23}$$

to compensate the angle error.

For the examble with an angle error of 1.5 arcmin following torque is necessary:

$$\hat{M}_{\text{CDangle}} = 27.5\frac{1.5\cdot 2\pi}{60\cdot 360\cdot 2}\cdot(2\pi\cdot 3)^2\,\text{Nm} = 2.1\,\text{Nm} \tag{24}$$

4.3 Torque needed to compensate the Backlash of a Gear Box

In many cases the power drive PD is a geared motor. The backlash $\Delta\varphi$ of a gear is noticeable when the torque or force is reversed. For a short time the gears do not have contact to each other and they can not transfer a torque or force. During this period the parallely working control drive CD has to supply the complete torque or force to the load.

To determine the load for the control drive CD a linear torque change with a jerk being constant ($\dot{\alpha} = \text{const}$) is adopted. This means that

$$M_{\text{set}} = t \cdot J_{\text{ges}} \cdot \dot{\alpha} = t \cdot (J_{\text{PD}} + J) \cdot \dot{\alpha} \tag{25}$$

During the torque reversal the control drive CD has to apply the torque change

$$\boxed{\hat{M}_{\text{CDbacklash}} = \dot{\alpha} \cdot (J_{\text{CD}} + J) \cdot \sqrt{\frac{\Delta\varphi}{\frac{1}{2}\left(\frac{J_{\text{PD}}+J}{J_{\text{PD}}} - 1\right) \cdot \dot{\alpha}}}} \tag{26}$$

for the maximum jerk $\dot{\alpha}$ of the machine.

To compensate a backlash of $\Delta\varphi = 15\text{arcmin}$ following torque and power is necessary:

$$\hat{M}_{\text{CDbacklash}} = 314.2 \cdot (0.032 + 4) \sqrt{\frac{\frac{15 \cdot 2 \cdot \pi}{60 \cdot 360}}{0.5 \cdot \left(\frac{27.5+4}{27.5} - 1\right) \cdot 314.2}}\ \text{Nm} = 17.5\ \text{Nm} \tag{27}$$

5 Control of the Double Drive

The control for the two drives working parallely is supposed to compensate the errors (torque deviation, backlash) of the power drive PD by using the exact working control drive CD. As a result the double drive has a linear transfer function. The double drive is integrated into the overall control system. The following investigation looks at a drive with speed control. The two drives CD and PD are assembled into the following overall drive (figure 5): PD and CD are torque followers. They receive the set values of the deviation to n_{set} or ω_{set}. PD and CD are each connected to the load by elasticities C_{PD} and C_{CD}. To control the load and speed the speed n_{PD} or ω_{PD} are measured at the power drive PD.

Five equations are neccessary for the two drives to be described. For the description of the drives the Laplace transformation [2] is used. In these equations $L(s)$ is an uncertain, slow variing transfer function of the power drive PD with errors.

- torque of the control drive CD:

$$\boxed{M_{\text{CD}} = V(s) \cdot R(s) \cdot (\omega_{\text{set}} - \omega_{\text{PD}}) = A(s) \cdot (\omega_{\text{set}} - \omega_{\text{PD}}) \text{ with } A(s) = V(s) \cdot R(s)} \tag{28}$$

- motion of the control drive CD:

$$\boxed{M_{\text{CD}} = s \cdot J_{\text{CD}} \cdot \omega_{\text{CD}} + \frac{1}{s} \cdot C_{\text{CD}} \cdot (\omega_{\text{CD}} - \omega_{\text{act}})} \tag{29}$$

- torque of the power drive PD:

$$\boxed{M_{\text{PD}} = L(s) \cdot (\omega_{\text{set}} - \omega_{\text{PD}})} \text{ as a simplified form of } M_{\text{PD}} = L_{\text{PD}}(s, \omega_{\text{set}}, \omega_{\text{PD}}) \tag{30}$$

- motion of the power drive PD:

$$\boxed{M_{\text{PD}} = s \cdot J_{\text{PD}} \cdot \omega_{\text{PD}} + \frac{1}{s} \cdot C_{\text{PD}} \cdot (\omega_{\text{PD}} - \omega_{\text{act}})} \tag{31}$$

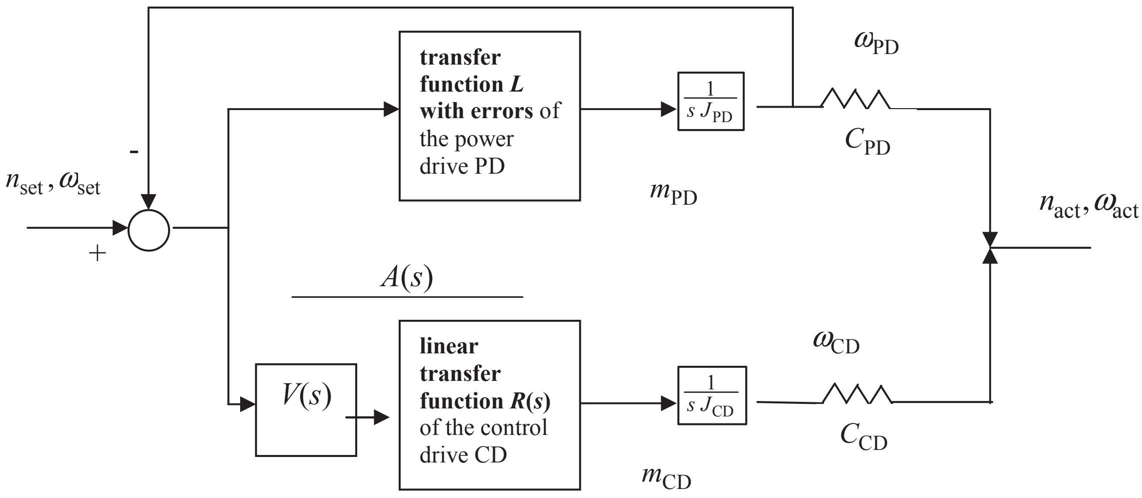

Figure 5: Control of a double drive consisting of a power drive PD and a control drive CD

► output of both drives:

$$\boxed{\omega_{act} = \frac{C_{PD}}{C_{PD}+C_{CD}}\omega_{PD} + \frac{C_{CD}}{C_{PD}+C_{CD}}\omega_{CD}} \tag{32}$$

In these equations one can find five unknown variables M_{CD} , M_{PD} , ω_{CD} , ω_{PD} and ω_{act} . With these equations the transfer function of the two drives can be calculated.

The first two equations (28) and (29) can be set equal. This eliminates M_{CD}:

$$A(s)\cdot(\omega_{set}-\omega_{PD}) = s\cdot J_{CD}\cdot\omega_{CD} + \frac{1}{s}\cdot C_{CD}\cdot(\omega_{CD}-\omega_{act}) \tag{33}$$

By setting the equations (30) and (31) equal M_{PD} can be eliminated:

$$L(s)\cdot(\omega_{set}-\omega_{PD}) = M_{PD} = s\cdot J_{PD}\cdot\omega_{PD} + \frac{1}{s}\cdot C_{PD}\cdot(\omega_{PD}-\omega_{act}) \tag{34}$$

The velocity ω_{CD} is eliminated from the above equations with the last equation (32) of the original ones by using

$$\omega_{CD} = \frac{C_{PD}+C_{CD}}{C_{CD}}\omega_{act} - \frac{C_{PD}}{C_{CD}}\omega_{PD} \tag{35}$$

The two equations (33) and (34) are solved for ω_{PD}:

$$\omega_{PD} = \frac{A(s)\cdot\omega_{set} - s\cdot J_{CD}\frac{C_{PD}+C_{CD}}{C_{CD}}\omega_{act} - \frac{1}{s}C_{PD}\cdot\omega_{act}}{A(s) - s\cdot J_{CD}\frac{C_{PD}}{C_{CD}} - \frac{1}{s}C_{PD}} \tag{36}$$

$$\omega_{PD} = \frac{L(s)\cdot\omega_{set} + \frac{1}{s}C_{PD}\cdot\omega_{act}}{L(s) + s\cdot J_{PD} + \frac{1}{s}C_{PD}} \tag{37}$$

By setting the equations (36) and (37) equal

$$\frac{A(s)\cdot\omega_{set} - s\cdot J_{CD}\frac{C_{CD}+C_{PD}}{C_{CD}}\omega_{act} - \frac{1}{s}C_{PD}\cdot\omega_{act}}{A(s) - s\cdot J_{CD}\frac{C_{PD}}{C_{CD}} - \frac{1}{s}C_{PD}} = \frac{L(s)\cdot\omega_{set} + \frac{1}{s}C_{PD}\cdot\omega_{act}}{L(s) + s\cdot J_{PD} + \frac{1}{s}C_{PD}} \tag{38}$$

the transfer function of the control system can be written as the following equation:

$$F(s) = \frac{n_{act}(s)}{n_{set}(s)} = \frac{\omega_{act}(s)}{\omega_{set}(s)} = \frac{s^2\cdot A(s)\cdot J_{PD} + s^2\cdot L(s)\cdot J_{CD}\frac{C_{PD}}{C_{CD}} + C_{PD}\cdot A(s) + C_{PD}\cdot L(s)}{\begin{array}{c} C_{PD}\cdot A(s) - s\frac{C_{PD}^2}{C_{CD}}J_{CD} + s^2\cdot L(s)\cdot J_{CD}\frac{C_{PD}+C_{CD}}{C_{CD}} \\ +L(s)\cdot C_{PD} + s^3\cdot J_{PD}\cdot J_{CD}\frac{C_{PD}+C_{CD}}{C_{CD}} + s\cdot J_{PD}\cdot C_{PD} \\ +s\cdot J_{CD}\cdot C_{PD}\frac{C_{PD}+C_{CD}}{C_{CD}} \end{array}} \tag{39}$$

This equation is transformed to the following simple expression:

$$F(s) = F_1(s)\cdot\frac{1 + F_2(s)\cdot L(s)}{1 + F_3(s)\cdot L(s)} \tag{40}$$

while

$$F_1(s) = \frac{s^2\cdot A(s)\cdot J_{PD}\cdot C_{CD} + C_{PD}\cdot C_{CD}\cdot A(s)}{s^3\cdot J_{PD}\cdot J_{CD}\left(C_{PD} + C_{CD}\right) + s\left(J_{PD} + J_{CD}\right)C_{PD}\cdot C_{CD} + C_{PD}\cdot C_{CD}\cdot A(s)} \tag{41}$$

$$F_2(s) = \frac{s^2\cdot J_{CD}\cdot C_{PD} + C_{PD}\cdot C_{CD}}{s^2\cdot A(s)\cdot J_{PD}\cdot C_{CD} + C_{PD}\cdot C_{CD}\cdot A(s)} \tag{42}$$

$$F_3(s) = \frac{s^2\cdot J_{CD}\left(C_{PD} + C_{CD}\right) + C_{PD}\cdot C_{CD}}{s^3\cdot J_{PD}\cdot J_{CD}\left(C_{PD} + C_{CD}\right) + s\left(J_{PD} + J_{CD}\right)C_{PD}\cdot C_{CD} + C_{PD}\cdot C_{CD}\cdot A(s)} \tag{43}$$

When the parameters of the drives are selected meaning that $F_2(s) = F_3(s)$, the function $L(s)$ of the power drive PD is eliminated from the transfer function (40).

In this case the transfer function of the total drive becomes the linear transfer function $F(s) = F_1(s)$. The nonlinear behavior of the power drive has no longer any influence on the transfer function, regardless of the properties the power drive has.

This is achieved when the control drive CD has the following transfer function:

$$A(s) = \frac{\begin{array}{c} s^4\cdot J_{PD}\cdot J_{CD}^2\cdot C_{PD}\left(C_{PD} + C_{CD}\right) \\ +s^2\cdot J_{CD}\left(2J_{PD}\cdot C_{PD}^2 + J_{CD}\cdot C_{PD}^2 + J_{PD}\cdot C_{PD}\cdot C_{CD}\right)C_{CD} \\ +\left(J_{PD} + J_{CD}\right)C_{PD}^2\cdot C_{CD}^2 \end{array}}{s^3\cdot J_{PD}\cdot J_{CD}\cdot C_{CD}\left(C_{PD} + C_{CD}\right) + s\left(J_{PD} + J_{CD}\right)C_{PD}\cdot C_{CD}^2} \tag{44}$$

That means the errors of the power drive PD are compensated completely by the appropriate choice of the transfer function $A(s)$ of the control drive CD.

The transfer function $A(s)$ of the CD is linear and completely independent of the properties of the PD. In the transfer function of the CD, one can find only the linear elasticity and inertia of the drives.

The nonlinear behavior of the power drive has no influence on the transfer function of the total drive. This offers the possibility to choose the behavior for a maximum efficiency and a small required power for the control drive.

The required power of the control drive can be minimized, when the power drive gets a feedforward to produce the needed torque in dependence of the required speed and torque of the load.

5.1 Simplifying the Transfer Function and stabilizing the Double Drive

The equation (44) mentioned above for $A(s)$ can not be implemented in reality. In particular, it has a counter surplus and is not stable. This means it is not applicable in this form. It can only be approximated.

The aproximation means a reduction of the frequency range so that the counter surplus will disappear. Therefore the consideration of the situation in the drives is analysed. In a specified frequenzy range $f = 0 \ldots f_{\max}$ the following relations are valid:

$$J_{PD} \cdot J_{CD}^2 \cdot C_{PD} \left(C_{PD} + C_{CD} \right) f^4 < \left(J_{PD} + J_{CD} \right) C_{PD}^2 \cdot C_{CD}^2 \tag{45}$$

$$J_{CD} \left(2J_{PD} \cdot C_{PD}^2 + J_{CD} \cdot C_{PD}^2 + J_{PD} \cdot C_{PD} \cdot C_{CD} \right) C_{CD} \cdot f^2 < \left(J_{PD} + J_{CD} \right) C_{PD}^2 \cdot C_{CD}^2 \tag{46}$$

$$J_{PD} \cdot J_{CD} \cdot C_{CD} \left(C_{PD} + C_{CD} \right) f^3 < \left(J_{PD} + J_{CD} \right) C_{PD} \cdot C_{CD}^2 \cdot f \tag{47}$$

$$\text{for } f < f_{\max} = \min \left\{ \begin{array}{c} \dfrac{1}{2\pi} \sqrt[4]{\dfrac{\left(J_{PD} + J_{CD} \right) C_{PD}^2 \cdot C_{CD}^2}{J_{PD} \cdot J_{CD}^2 \cdot C_{PD} \left(C_{PD} + C_{CD} \right)}} \\[2em] \dfrac{1}{2\pi} \sqrt{\dfrac{\left(J_{PD} + J_{CD} \right) C_{PD}^2 \cdot C_{CD}^2}{J_{CD} \left(2J_{PD} \cdot C_{PD}^2 + J_{CD} \cdot C_{PD}^2 + J_{PD} \cdot C_{PD} \cdot C_{CD} \right) C_{CD}}} \\[2em] \dfrac{1}{2\pi} \sqrt{\dfrac{\left(J_{PD} + J_{CD} \right) C_{PD} \cdot C_{CD}^2 \cdot}{J_{PD} \cdot J_{CD} \cdot C_{CD} \left(C_{PD} + C_{CD} \right)}} \end{array} \right\} \tag{48}$$

With these ideas the transfer function of the control drive CD can be simplified by neglecting the smaller parts in numerator and denominator:

$$A(s) \approx A_a(s) = \frac{\left(J_{PD} + J_{CD} \right) C_{PD}^2 \cdot C_{CD}^2}{s \left(J_{PD} + J_{CD} \right) C_{PD} \cdot C_{CD}^2} = \frac{C_{PD}}{s} \quad \text{for} \quad f < f_{\max} \tag{49}$$

This means a normal integral controller for the control drive CD is able to compensate the errors of the power drive PD in a specified frequency range $f = 0 \ldots f_{\max}$.

With $A_a(s) = \frac{C_{PD}}{s}$ the transfer function $F(s)$ from equation (40) becomes the following expression with $F_2(s) \approx F_3(s)$:

$$F(s) \approx F_{1a}(s) = \frac{s^2 \cdot J_{PD} \cdot C_{CD} \cdot A_a(s) + C_{PD} \cdot C_{CD} \cdot A_a(s)}{s^3 \cdot J_{PD} \cdot J_{CD} \left(C_{PD} + C_{CD} \right) + s \left(J_{PD} + J_{CD} \right) C_{PD} \cdot C_{CD} + C_{PD} \cdot C_{CD} \cdot A_a(s)}$$

$$= \frac{s^2 \cdot J_{PD} \cdot C_{PD} \cdot C_{CD} + C_{PD}^2 \cdot C_{CD}}{s^4 \cdot J_{PD} \cdot J_{CD} \left(C_{PD} + C_{CD} \right) + s^2 \left(J_{PD} + J_{CD} \right) C_{PD} \cdot C_{CD} + C_{PD}^2 \cdot C_{CD}} \tag{50}$$

This function is not stable because the denominator does not fullfill the Hurwitz criteria or Routh criteria saying that all the coefficients a_i of the denominator polynom $s^4 a_4 + s^3 a_3 + s^2 a_2 + s a_1 + a_0$ must not be equal 0 and must have the same sign.

In fact two coefficients of the denominator polynom are 0: $a_1 = 0$ and $a_3 = 0$. This means the function is not stable.

However, the system is at its limit of stability because in the mechanical system for simplification damping is not considered. With the coefficients $a_1 = 0$ and $a_3 = 0$ this system can not work.

To stabilize the system the transfer function $A_{\mathrm{a}}(s)$ needs an additional propotional and differential part.

The following function A_{b} has both parts. Additionaly a denominator $1 + T_{\mathrm{A}}$ eliminates the counter surplus.

$$\boxed{A_{\mathrm{b}}(s) = \frac{C_{\mathrm{PD}}}{1 + s \cdot T_{\mathrm{A}}} \left(\frac{1}{s} + a_{\mathrm{p}} + s \cdot a_{\mathrm{d}} \right)} \tag{51}$$

For low frequencies both functions are approximatly the same:

$$A_{\mathrm{b}}(s) \approx A_{\mathrm{a}}(s) \tag{52}$$

for

$$\boxed{f_{\mathrm{maxCD}} < \min\left(\frac{1}{2\pi \cdot T_{\mathrm{A}}}, \frac{1}{2\pi \cdot a_{\mathrm{p}}}, \frac{1}{2\pi\sqrt{|a_{\mathrm{d}}|}} \right)} \tag{53}$$

With $A_{\mathrm{b}}(s)$ the transfer function $F(s)$ from equation (40) becomes the following expression with $F_2(s) \approx F_3(s)$:

$$F(s) \approx F_{1\mathrm{b}}(s) = \frac{\left(s^2 J_{\mathrm{PD}} \cdot C_{\mathrm{CD}} + C_{\mathrm{PD}} \cdot C_{\mathrm{CD}}\right) A_{\mathrm{b}}(s)}{s^3 \cdot J_{\mathrm{PD}} \cdot J_{\mathrm{CD}} \left(C_{\mathrm{PD}} + C_{\mathrm{CD}}\right) + s \left(J_{\mathrm{PD}} + J_{\mathrm{CD}}\right) C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} + C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \cdot A_{\mathrm{b}}(s)}$$

$$= \frac{\begin{gathered} s^4 \cdot J_{\mathrm{PD}} \cdot C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{d}} + s^3 \cdot J_{\mathrm{PD}} \cdot C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{p}} \\ + s^2 \left(J_{\mathrm{PD}} \cdot C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} + \cdot C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{d}}\right) \\ + s \cdot C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{p}} + C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \end{gathered}}{\begin{gathered} s^5 \cdot J_{\mathrm{PD}} \cdot J_{\mathrm{CD}} \left(C_{\mathrm{PD}} + C_{\mathrm{CD}}\right) T_{\mathrm{A}} + s^4 \cdot J_{\mathrm{PD}} \cdot J_{\mathrm{CD}} \left(C_{\mathrm{PD}} + C_{\mathrm{CD}}\right) \\ + s^3 \left(J_{\mathrm{PD}} + J_{\mathrm{CD}}\right) C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \cdot T_{\mathrm{A}} + s^2 \left(J_{\mathrm{PD}} + J_{\mathrm{CD}} + C_{\mathrm{PD}} \cdot a_{\mathrm{d}}\right) C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \\ + s \cdot C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{p}} + C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \end{gathered}} \tag{54}$$

For a stable operation the demoniator polynom has to have eigenvalues with a negative real part. The denominator polynom of eqation (54) is

$$s^5 \cdot a_5 + s^4 \cdot a_4 + s^3 \cdot a_3 + s^2 \cdot a_2 + s \cdot a_1 + a_0 \tag{55}$$

while

$$a_0 = C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \tag{56}$$

$$a_1 = C_{\mathrm{PD}}^2 \cdot C_{\mathrm{CD}} \cdot a_{\mathrm{p}} \tag{57}$$

$$a_2 = \left(J_{\mathrm{PD}} + J_{\mathrm{CD}} + C_{\mathrm{PD}} \cdot a_{\mathrm{d}}\right) C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \tag{58}$$

$$a_3 = \left(J_{\mathrm{PD}} + J_{\mathrm{CD}}\right) C_{\mathrm{PD}} \cdot C_{\mathrm{CD}} \cdot T_{\mathrm{A}} \tag{59}$$

$$a_4 = J_{\mathrm{PD}} \cdot J_{\mathrm{CD}} \left(C_{\mathrm{PD}} + C_{\mathrm{CD}}\right) \tag{60}$$

$$a_5 = J_{\mathrm{PD}} \cdot J_{\mathrm{CD}} \left(C_{\mathrm{PD}} + C_{\mathrm{CD}}\right) T_{\mathrm{A}} \tag{61}$$

Table 3: Routh scheme for $n = 5$

a_5	a_3	a_1
a_4	a_2	a_0
$A_1 = \frac{a_4 a_3 - a_5 a_2}{a_4}$	$B_1 = \frac{a_4 a_1 - a_5 a_0}{a_4}$	$C_1 = 0$
$A_2 = \frac{A_1 a_2 - a_4 B_1}{A_1}$	$B_2 = \frac{A_1 a_0 - a_4 c_1}{A_1}$	$C_2 = 0$
$A_3 = \frac{A_2 B_1 - A_1 B_2}{A_2}$	$B_3 = \frac{A_2 C_1 - A_1 C_2}{A_2} = 0$	$C_3 = 0$
$A_4 = \frac{A_3 B_2 - A_2 B_3}{A_3}$	$B_4 = \frac{A_3 C_2 - A_2 C_3}{A_3} = 0$	$C_4 = 0$

To check the stability the Routh criteria is used [1]. The results for the examble are shown in table 1.

The first condition of the Routh criteria demands that all coefficients exist and have the same sign. In this case all coefficients must be positive. This leads to the following conditions for a_p, a_d and T_A:

$$a_0 = C_\text{PD}^2 \cdot C_\text{CD} > 0 \checkmark \quad (62)$$

$$a_1 = C_\text{PD}^2 \cdot C_\text{CD} \cdot a_\text{p} \overset{!}{>} 0 \Rightarrow \boxed{a_\text{p} > 0} \quad (63)$$

$$a_2 = (J_\text{PD} + J_\text{CD} + C_\text{PD} \cdot a_\text{d}) C_\text{PD} \cdot C_\text{CD} \overset{!}{>} 0 \Rightarrow \boxed{a_\text{d} > -\frac{J_\text{PD} + J_\text{CD}}{C_\text{PD}}} \quad (64)$$

$$a_3 = (J_\text{PD} + J_\text{CD}) C_\text{PD} \cdot C_\text{CD} \cdot T_\text{A} \overset{!}{>} 0 \Rightarrow \boxed{T_\text{A} > 0} \quad (65)$$

$$a_4 = J_\text{PD} \cdot J_\text{CD} (C_\text{PD} + C_\text{CD}) > 0 \checkmark \quad (66)$$

$$a_5 = J_\text{PD} \cdot J_\text{CD} (C_\text{PD} + C_\text{CD}) T_\text{A} \overset{!}{>} 0 \Rightarrow T_\text{A} > 0 \quad (67)$$

The second condition of the Routh criteria demands that all test functions are positive: $A_i > 0$ for $i = 1 \ldots 4$. The values A_i are calculated according to the Routh scheme (see table 3). Further conditions for the coefficients a_p, a_d and T_A are evaluated from these demands.

The first test function A_1 provides:

$$A_1 = \frac{a_4 \cdot a_3 - a_5 \cdot a_2}{a_4} = -C_\text{PD}^2 \cdot C_\text{CD} \cdot T_\text{A} \cdot a_\text{d} \Rightarrow \boxed{a_\text{d} < 0} \quad (68)$$

The second function A_2 leads to another condition for a_d:

$$A_2 = \frac{A_1 \cdot a_2 - a_4 \cdot B_1}{A_1} \quad \text{with} \quad B_1 = \frac{a_4 \cdot a_1 - a_5 \cdot a_0}{a_4} = C_\text{PD}^2 \cdot C_\text{CD}^2 \left(a_\text{p} - T_\text{A}\right) \quad (69)$$

$$A_2 = \frac{\begin{matrix} -J_\text{PD} \cdot C_\text{PD} \cdot C_\text{CD} \cdot T_\text{A} \cdot a_\text{d} - J_\text{CD} \cdot C_\text{PD} \cdot C_\text{CD} \cdot T_\text{A} \cdot a_\text{d} \\ -C_\text{PD}^2 \cdot C_\text{CD} \cdot T_\text{A} \cdot a_\text{d}^2 - J_\text{PD} \cdot J_\text{CD} (C_\text{PD} + C_\text{CD}) \left(a_\text{p} - T_\text{A}\right) \end{matrix}}{-T_\text{A} \cdot a_\text{d}} \quad (70)$$

$$\Rightarrow \boxed{a_\text{d} < a_\text{d2}} \quad (71)$$

while

$$\boxed{a_\text{d2} = -\frac{J_\text{PD} + J_\text{CD}}{c_\text{PD}} + \sqrt{\left(\frac{J_\text{PD} + J_\text{CD}}{c_\text{PD}}\right)^2 - \frac{J_\text{PD} \cdot J_\text{CD} (C_\text{PD} + C_\text{CD}) \left(a_\text{p} - T_\text{A}\right)}{C_\text{PD}^2 \cdot C_\text{CD} \cdot T_\text{A}}}} \quad (72)$$

In this case only the positive value of the squareroot is used because the negative value is fullfilled by the equation (64). The squareroot needs a positive argument to get real values for $a_{d1,2}$. This leads to the following condition for a_p:

$$\left(\frac{J_{PD}+J_{CD}}{C_{PD}}\right)^2-\frac{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)\left(a_p-T_A\right)}{C_{PD}^2\cdot C_{CD}\cdot T_A}\geq 0 \tag{73}$$

$$\Rightarrow \boxed{a_p\leq\frac{\left(J_{PD}+J_{CD}\right)^2 C_{CD}\cdot T_A}{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}+T_A} \tag{74}$$

The third function A_3 gives another condition for a_p:

$$A_3=\frac{A_2\cdot B_1-A_1\cdot B_2}{A_2}\quad\text{with}\quad C_1=0\,,\quad B_2=\frac{A_1\cdot a_0-a_4\cdot C_1}{A_1}=a_0=C_{PD}^2\cdot C_{CD} \tag{75}$$

$$A_3=C_{PD}^2\cdot C_{CD}\left(a_p-T_A\right)-\frac{C_{PD}^4\cdot C_{CD}^2\cdot T_A^2\cdot a_d^2}{\begin{array}{c}-\left(J_{PD}+J_{CD}\right)\cdot C_{PD}\cdot C_{CD}\cdot T_A\cdot a_d-C_{PD}^2\cdot C_{CD}\cdot T_A\cdot a_d^2\\ -J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)\left(a_p-T_A\right)\end{array}} \tag{76}$$

$$\Rightarrow \boxed{a_{p1}<a_p<a_{p2}} \tag{77}$$

while

$$a_{p1,2}=-\frac{\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot T_A\cdot a_d-2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)T_A+C_{PD}^2\cdot C_{CD}\cdot T_A\cdot a_d^2}{2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}\pm\sqrt{\left(\frac{\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot T_A\cdot a_d-2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)T_A+C_{PD}^2\cdot C_{CD}\cdot T_A\cdot a_d^2}{2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}\right)^2-\frac{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)T_A^2-\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot T_A^2\cdot a_d}{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}} \tag{78}$$

$$\boxed{a_{p1,2}=T_A\cdot\left(-\frac{\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot a_d-2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)+C_{PD}^2\cdot C_{CD}\cdot a_d^2}{2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}\pm\sqrt{\left(\frac{\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot a_d-2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)+C_{PD}^2\cdot C_{CD}\cdot a_d^2}{2J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}\right)^2-\frac{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)-\left(J_{PD}+J_{CD}\right)C_{PD}\cdot C_{CD}\cdot a_d}{J_{PD}\cdot J_{CD}\left(C_{PD}+C_{CD}\right)}}\right)} \tag{79}$$

The fourth function A_4 is allways positive:

$$A_4=\frac{A_3\cdot B_2-A_2\cdot B_3}{A_3}\quad\text{with}\quad B_3=0 \tag{80}$$

$$\Rightarrow A_4=B_2=C_{PD}^2\cdot C_{CD}>0\ \checkmark \tag{81}$$

The allowable limits for the parameters a_p, a_d and T_A for the examble are shown in table 1.

The control of the drive system with the simplified transfer function compensates the errors of the power drive PD part wise depending on the frequency.

This advantage is especially usefull, when the control drive CD only needs a small power in relation to the power drive PD. In this case the complete drive system has lower costs than a high precision drive for the complete power.

6 Simulation of a Double Drive with 37 kW

The concept described above is simulated on a drive with $n = 180\ ^1/\mathrm{min}$ and $M = 1963$ Nm to show the effect and stability of the control. The drive works as a synchronizing drive and must follow small set-point changes as closely as possible at a constant average speed. The power drive PD is an asynchronous motor with inverter and gear box. The control drive CD is a gearless permanent-magnet servo motor with servo inverter (further data see figure 2 and table 1).

The set speed is shown in figure 6. First the drive is accelerated from standstill to a speed of 180 $^1/\mathrm{min}$. During the constant speed phase the drive has to correct its angle by 1.8°. Therefore the set speed is increased form 180 $^1/\mathrm{min}$ to 183 $^1/\mathrm{min}$ during a time of 0.1 s. Afterwards the speed is reduced back to 180 $^1/\mathrm{min}$ during a time of also 0.1 s.

The power drive has a feedforward from the set speed to produce the required torque for accelerating the inertia. The required torque is $J_\mathrm{PD} \cdot s \cdot \omega_\mathrm{set}$. The coresponding term is integrated into the equation for $L_\mathrm{PD}(s)$

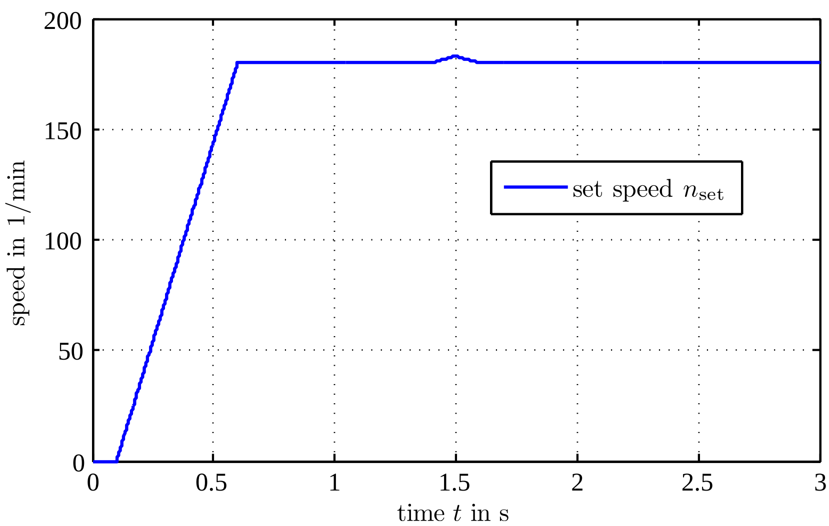

Figure 6: Set speed n_set for the simulation of power drive with 37 kW, 180 $^1/\mathrm{min}$

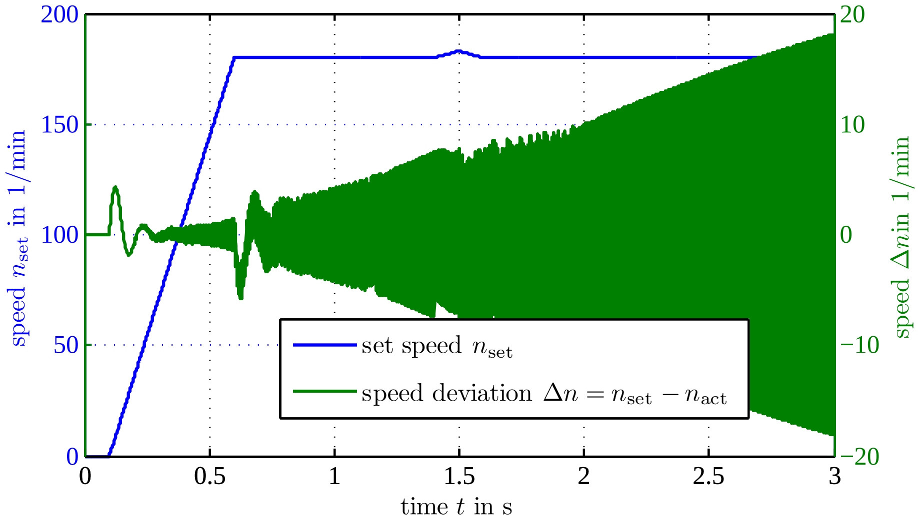

Figure 7: Speed deviation $\Delta n = n_{set} - n_{act}$ for linear power drive with 37 kW, 180 1/min, $f_0 = 5$ Hz

6.1 Simulation of the Power Drive alone

In the first analysis the power drive PD alone accelerates and runs the complete drive. The power drive is simulated with the function

$$M_{PD} = L_{PD}(s, \omega_{set}, \omega_{PD}) = L_a(s)\,(V_{PD} \cdot (\omega_{set} - \omega_{PD}) + s \cdot J_{PD} \cdot \omega_{set}) \tag{82}$$

while

$$L_a(s) = \frac{1}{1 + s \cdot T_{PD}} = \frac{1}{1 + s \cdot 0.03183\,\mathrm{s}} \quad , \quad V_{PD} = 3000\,\frac{\mathrm{Nm}}{\mathrm{rad/s}} \quad , \quad J_{PD} = 27.5\,\mathrm{kg\,m}^2 \tag{83}$$

With this cutoff frequency

$$f_{0PD} = \frac{1}{2\pi \cdot T_{PD}} = 5\,\mathrm{Hz} \tag{84}$$

the gain of $V_{PD} = 3000\,\frac{\mathrm{Nm}}{\mathrm{rad/s}}$ is usefull for an acceptable damping for the power drive PD alone. The control drive does not work. Its torque is $M_{CD} = 0$.

In figure 7 the result of the simulation for the speed deviation $\Delta n = n_{set} - n_{act}$ is shown. When the set speed n_{set} changes the actual speed n_{act} shows oszillations. The drive ist not stable and the oszillations grow with time. These oszillations have their origin in the elastic coupling C_{PD} without damping between the power drive PD and the load.

The angle deviation $\Delta\varphi = \varphi_{set} - \varphi_{act}$ is shown in figure 8. During the angle correction of 1.8° the angle oszillation is 0.25°.

The result gets worser when an sinusoidal torque distortion

$$M_{sin} = \hat{M}_{sin} \cdot \sin(2\pi \cdot f_{sin} \cdot t) = 150\,\mathrm{Nm} \cdot \sin(2\pi \cdot 6\,\mathrm{Hz} \cdot t) \tag{85}$$

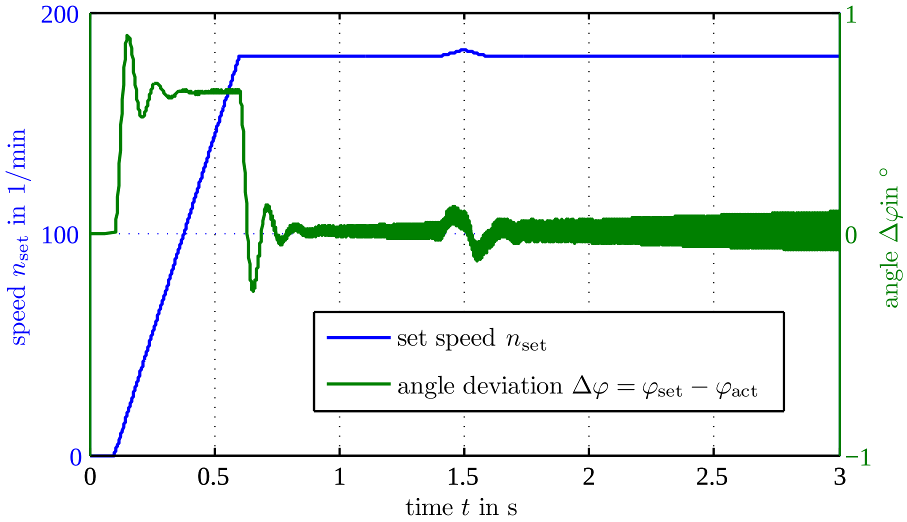

Figure 8: Angle deviation $\Delta\varphi = \varphi_{set} - \varphi_{act}$ for linear power drive with 37 kW, 180 $^{1}/_{min}$, $f_0 = 5$ Hz

and a torque dead zone of $M_{dz} = \pm 100$ Nm is added. Figure 9 shows the torque of the power drive. During the acceleration the torque is about 1000 Nm. During the whole period the oszillations of the sinusoidal distortion with the dead zone are visible. The angle behavior under the same condition is shown in figure 10. The angle oszillation is about 0.5°.

The diagram also shows the oszillations due to the mechanics which can not be controled alone by the power drive. With this behavior the power drive alone is not usefull for many precise applications.

6.2 Simulation of the Double Drive

Figures 11, 12 and 13 show the results for torque, speed and angle with the double drive for the same set speed n_{set} as in figure 6. The control drive has the parameters

$$a_p = 0.005 \text{ s} \tag{86}$$

$$a_d = -0.00005 \text{ s}^2 \tag{87}$$

$$C_{PD} = 200\,000 \, \frac{\text{Nm}}{\text{rad}} \tag{88}$$

$$T_A = 0.001 \text{ s} \tag{89}$$

and the transfer function is

$$A_b = \frac{200\,000 \, \frac{\text{Nm}}{\text{rad}}}{1 + s \cdot 0.001 \text{ s}} \left(\frac{1}{s} + 0.005 \, \frac{1}{\text{s}} - s \cdot 0.00005 \, \frac{1}{\text{s}^2} \right) \tag{90}$$

The parameters a_p, a_d and T_A are in aggreement with the demands of section 5. The allowable limits are shown in table 1.

This control drive with the transfer function A_b compensates with its torque M_{CD} (figure 11) most of the errors of the power drive.

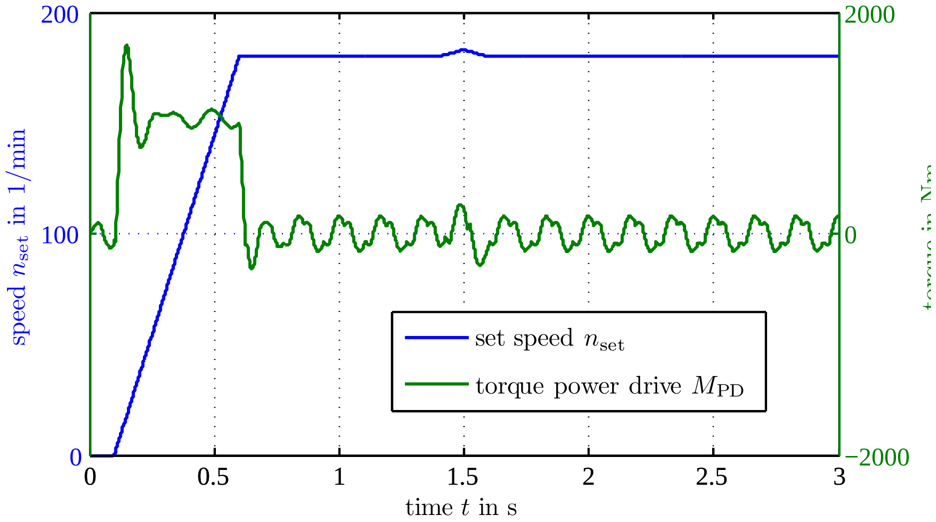

Figure 9: Torque M_{PD} of power drive with 37 kW with additional sinusoidal torque $\hat{M}_{\mathrm{sin}} = 150$ Nm, dead zone $M_{\mathrm{dz}} = \pm 100$ Nm, 180 $^{1}/\mathrm{min}$, $f_0 = 5$ Hz

In figure 12 is shown that the speed difference Δn now is less than 2 $^{1}/\mathrm{min}$ during the angle correction. Most of the time the deviation is less than 1 $^{1}/\mathrm{min}$. Only during the acceleration the speed deviation is higher because the control drive is limited to 400 Nm. The oszillations of the mechanics are damped and the whole drive works stable during the complete period.

Figure 13 shows the coresponding results for the angle difference $\Delta\varphi = \varphi_{\mathrm{set}} - \varphi_{\mathrm{act}}$. Now the difference is $\Delta\varphi < 0.05°$ during the time of angle correction. Only during the acceleration higher differences are seen because the torque of the control drive is limited to 400 Nm.

So far the results show the effectiveness of the control drive CD to compensate errors of the power drive PD by a suitable controller. The complete drive is stabilized by the control drive. The angle errors are reduced to less than 20% of the original errors of the power drive.

The simulation is done with a simplified mechanic. The load is completely neglected.

Further studies are needed to demonstrate the practical feasibility with real mechanics and loads. These studies should deliver the optimization of the controller to compensate the errors. In particular the robustness to parameter variations and the constructive realization should be investigated in the future. Other points are the costs for a double drive in relation to the actual solutions.

7 Summary

The mechatronic drive system for geared motors with a double drive improve the accuracy of the overall drive. In the calculation of an actual drive is shown that the control drive CD needs only about 20% of the power of the power drive PD to compensate the errors to a great extent. The stability of the double drive is investigated by a numerical simulation.

To demonstrate the practical feasibility and the optimization of the controller to compensate the errors further studies are needed. In particular, the robustness to parameter variations and the constructive

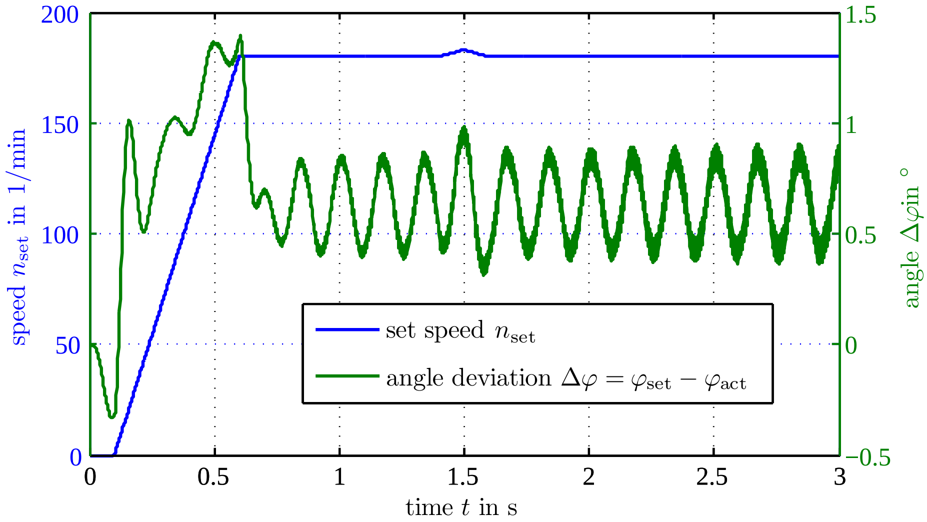

Figure 10: Angle deviation $\Delta\varphi = \varphi_{set} - \varphi_{act}$ for power drive with 37 kW with additional sinusoidal torque $\hat{M}_{sin}$ = 150 Nm, dead zone M_{dz} = ±100 Nm, 180 1/min, f_0 = 5 Hz

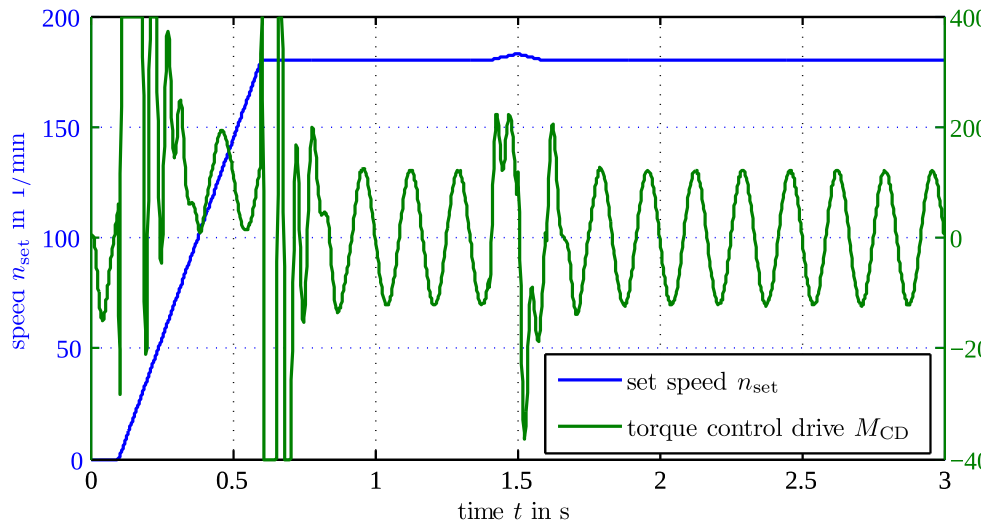

Figure 11: Torque M_{CD} of control Drive with limit ±400 Nm for power drive with 37 kW with additional sinusoidal torque $\hat{M}_{sin}$ = 150 Nm, dead zone M_{dz} = ±100 Nm, 180 1/min, f_0 = 5 Hz

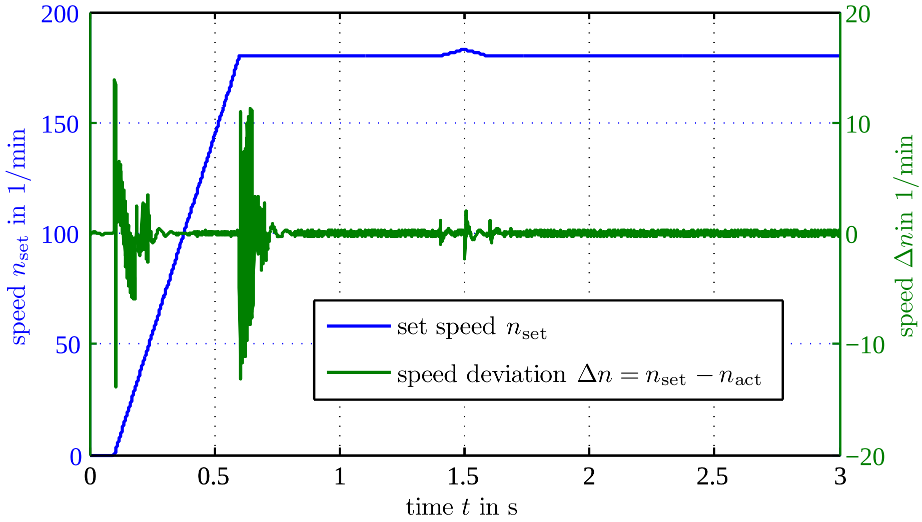

Figure 12: Speed deviation $\Delta n = n_{set} - n_{act}$ with control Drive with limit ±400 Nm and power drive with 37 kW with additional sinusoidal torque $\hat{M}_{sin} = 150$ Nm, dead zone $M_{dz} = \pm 100$ Nm, 180 1/min, $f_0 = 5$ Hz

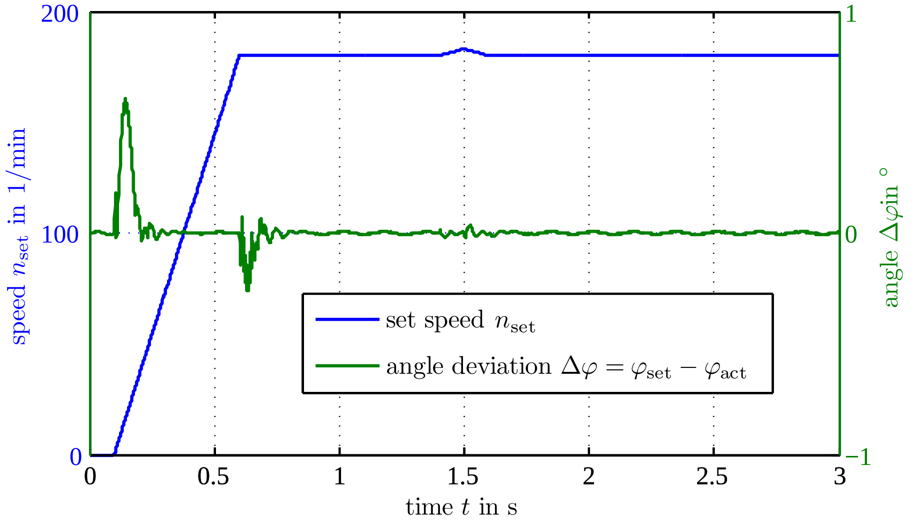

Figure 13: Angle deviation $\Delta\varphi = \varphi_{set} - \varphi_{act}$ with control Drive with limit ±400 Nm and power drive with 37 kW with additional sinusoidal torque $\hat{M}_{sin} = 150$ Nm, dead zone $M_{dz} = \pm 100$ Nm, 180 1/min, $f_0 = 5$ Hz

realization are focused. The simulation should be expanded to consider all elements of the drive including the behavior of the load.

So far in investigations is shown a way to realize compact and high-precision drives with low cost using standard components.

References

[1] Lunze: Regelungstechnik 1. 7. Auflage, Routh criteria p. 395–397, Springer (2008)

[2] Schumacher, Leonhard: Grundlagen der Regelungstechnik. Technische Universität Braunschweig, Institut für Regelungstechnik, Laplace transformation p. 41 pp. (2008)

[3] Fräger: Mechatronisches Antriebskonzept für Getriebemotoren – hochgenaue geregelte Umrichter-Antriebe mit geringen Kosten. Conference SPS–IPC–Drives, Nuremberg (2007)

[4] Fräger: Mechatronisches Antriebskonzept für Getriebemotoren – hochgenaue geregelte Umrichter-Antriebe mit geringen Kosten. Computer & Automation (2008)